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Infrared Radiation in the Atmosphere*

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The infrared radiation flux in the atmosphere is determined by the distribution of three gases—water vapor, carbon dioxide, and ozone. The general equations for the transfer of radiation in a gas are derived. These equations are solved for four simple model atmospheres that illustrate various features of the more complicated results for the earth's atmosphere. The atmospheric infrared radiation flux can be calculated from laboratory absorption measurements. The results of such calculations are discussed in detail for the frequency ranges that are influenced by the carbon dioxide and ozone bands.

1. INTRODUCTION

THREE gases—water vapor, carbon dioxide, and ozone—occur naturally in concentrations of less than a few percent in the earth's atmosphere and yet they determine the infrared radiation flux. The climate at the surface of the earth and the variation of temperature with height would be considerably different should the concentration or distribution of any of these gases change appreciably.

From the surface of the earth up to heights of 80 km or more the temperature is usually in the range from 200 to 320°K. The blackbody radiation that corresponds to these temperatures is largely in the spectral region from 5 to 100 microns. In order to influence the infrared flux in the atmosphere, a gas must have an absorption band in this range of frequencies. None of the three most common atmospheric gases—nitrogen, oxygen, and argon—have such absorption bands. On the other hand, ozone has an appreciable absorption coefficient from 9–10μ, carbon dioxide from 12–18μ, and water vapor above 20μ and to a lesser extent at smaller wavelengths. Some very rare constituents of the atmosphere, such as CO, NO, N_2O, and CH_4 also have absorption bands in this spectral range, but they have no appreciable influence on the infrared flux because of the very small number of such molecules that exist in the atmosphere.

Only radiation processes in the atmosphere are considered in this article. It should be emphasized that radiation by itself does not determine the temperature at a given altitude. From the surface of the earth up to the tropopause such processes as the adiabatic cooling of the gas during convective processes, turbulent heat transfer, and the absorption and release of the latent heat of water vapor must also be considered. However, in the stratosphere, the absorption and emission of radiation probably determine the average temperature distribution, although this may be modified at certain times

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by convective processes. The slow rise in temperature from 20 to 50 km is caused by the absorption of ultraviolet radiation from the sun by ozone. Above 20 km the infrared radiation from the carbon dioxide, ozone and water vapor acts to cool the atmosphere. Above 50 km the ultraviolet absorption of ozone is no longer large enough to balance the infrared cooling and the temperature decreases with height. Above 80 km the temperature again increases with height as ultraviolet light is absorbed by dissociated molecules.

A review of some of the methods for the calculation of the infrared radiation flux is given in this article. In Sec. 2 the general equations are derived that determine the radiative flux at a given height. Some solutions of these equations are given in Sec. 3 for several simple atmospheric models. These simple models illustrate many of the same features that occur in the much more complicated problem of the earth's atmosphere. The numerical results for some of these model atmospheres are given in Sec. 4 and their physical meaning is discussed. A method for the determination of the infrared flux in the atmosphere from laboratory absorption measurements is discussed in Sec. 5. Finally the results of calculations of the infrared flux in the earth's atmosphere are presented in Secs. 6 and 7. The spectral region from 12–18μ which is influenced by carbon dioxide is discussed in Sec. 6; the region from 9–10.1μ which is influenced by ozone is discussed in Sec. 7.

2. DERIVATION OF EQUATIONS FOR RADIATIVE FLUX

In this section the general solution is derived for the radiative flux at any height in the atmosphere. Some simplified forms of these solutions are given for particular temperature distributions and when certain other conditions are satisfied.

Consider first the upward component of the radiative flux, $I\!\uparrow$, which makes an angle θ with the vertical. The change in the upward flux, $dI\!\uparrow$, in a horizontal layer in the atmosphere of thickness dz is given by the expression

$$dI\!\uparrow = -kI\!\uparrow \sec\theta \rho_r dz + kI_b \sec\theta \rho_r dz, \qquad (1)$$

where k is the absorption coefficient, I_b is the blackbody radiation flux determined from the Planck radiation law, ρ_r is the density of the radiating gas, and z is the height above the surface of the earth. The first term on the right of Eq. (1) represents the energy absorbed from the incoming beam by the absorbing gas in the infinitesimal layer; the second term represents the energy that is emitted by this same layer of gas and is added to the original beam. A consistent set of units in the cgs system is to have $I\!\uparrow$ and I_b expressed in ergs cm^{-2} sec^{-1} and k in cm^2 g^{-1}.

Equation (1) assumes that Kirchhoff's law is valid so that both the emission and absorption of radiation can be expressed in terms of the absorption coefficient. This assumption is certainly valid at any altitude considered in this article (less than 75 km). However, at very great altitudes the number of molecules in excited states may be less than the number prescribed by the Boltzmann distribution because of the infrequent molecular collisions and the action of the radiation field.

The radiation at only a single frequency is determined by Eq. (1); k in particular is the appropriate absorption coefficient at this frequency. In the atmosphere k is usually a very rapidly varying function of frequency, since the absorption and emission of radiation is determined by thousands of spectral lines in the molecular bands of water vapor, carbon dioxide, and ozone. In this article it is assumed that the pressure and temperature at a given altitude are known and the radiation flux is then calculated for this distribution. Since the absorption coefficient of a spectral line varies with both pressure and temperature and these quantities are assumed to be known at a given altitude, k may be considered to be a function of the altitude and of the frequency. Similarly, since the blackbody intensity depends on both frequency and temperature, I_b in Eq. (1) may be considered to be a function of the frequency and altitude.

The introduction of a new independent variable and of a transmission function greatly simplifies the solution of Eq. (1). The mass of radiating gas per unit area is

$$u = \int_z^\infty \rho_r dz = \int_z^\infty c\rho dz, \qquad du = -\rho_r dz, \qquad (2)$$

where ρ is the total density of the atmosphere at the height z and the fractional concentration c of the radiating gas is the ratio of ρ_r to ρ. In most of the following work, u is taken as the independent variable. Note that u is zero at the top of the atmosphere and increases as the height above the surface of the earth decreases. Every layer in the atmosphere can be assigned a unique value of u, but according to Eq. (2) this value is different at a given altitude for each radiating gas and also varies with the distribution of the radiating gas with height.

The definition of the transmission function, $\tau(u_0,u_1)$, between two heights corresponding to the values u_0 and u_1 for the mass of radiating gas per unit area is that

$$\tau(u_0,u_1)=\exp\left[-\sec\theta\int_{u_0}^{u_1}kdu\right],\quad u_0<u_1. \quad (3)$$

The transmission function is only defined when $u_0<u_1$. When the absorption is very small, k is nearly zero and the transmission function is almost unity. Conversely when the absorption is large, k is a large number and the transmission function is nearly zero.

The change in the upward flux can now be written from Eqs. (1) and (2) as

$$dI\uparrow=kI\uparrow\sec\theta du-kI_b\sec\theta du. \quad (4)$$

The absorption coefficient and blackbody intensity are now considered as a function of u, since a unique value of u is associated with each altitude and the pressure and temperature are assumed to be known as a function of altitude. These same quantities also vary with frequency, but Eq. (4) refers to a single frequency only.

The general solution of Eq. (4) can be written in terms of the transmission function as

$$I\uparrow(u)=I\uparrow(u_1)\tau(u,u_1)+\sec\theta\int_u^{u_1}k(v)I_b(v)\tau(u,v)dv, \quad (5)$$

where u_1 is the value of u at the lower boundary (usually the surface of the earth). It is easily verified that this is the appropriate solution by substitution in the original differential equation. The physical meaning of this solution is evident; it states that the upward flux at a given height, u, is equal to the upward flux at the lower boundary layer, u_1, multiplied by the transmission between u and u_1 plus the blackbody flux emitted by each layer of gas between u and u_1 times the transmission between u and the height of the emitting layer.

For many purposes it is more convenient to express the solution in a form that does not explicitly depend on the absorption coefficient, since this is a very rapidly varying function of frequency in a band spectrum. This can be done with a single integration by parts if we notice from the definition of the transmission function, Eq. (3), that the integral of $\sec\theta k(v)\tau(u,v)$ with respect to v is $-\tau(u,v)$. Thus

$$I\uparrow(u)=I\uparrow(u_1)\tau(u,u_1)+I_b(u)-I_b(u_1)\tau(u,u_1)+\int_u^{u_1}\tau(u,v)\frac{dI_b(v)}{dv}dv. \quad (6)$$

Entirely similar equations apply for the downward radiation flux at an angle θ to the vertical. The angle θ is always chosen so that $0\leqq\theta\leqq\frac{1}{2}\pi$. The differential equation is

$$dI\downarrow=-kI\downarrow\sec\theta du+kI_b\sec\theta du. \quad (7)$$

The general solution is

$$I\downarrow(u)=I\downarrow(u_0)\tau(u_0,u)+\sec\theta\int_{u_0}^{u}k(v)I_b(v)\tau(v,u)dv, \quad (8)$$

where u_0 is the value of u at the upper boundary. From one integration by parts this equation can be transformed into

$$I\downarrow(u)=I\downarrow(u_0)\tau(u_0,u)+I_b(u)-I_b(u_0)\tau(u_0,u)-\int_{u_0}^{u}\tau(v,u)\frac{dI_b(v)}{dv}dv. \quad (9)$$

Usually the upper boundary is taken at the top of the atmosphere so that $u_0=0$. Since the incoming infrared flux beyond 5μ that reaches the earth from the sun is extremely small compared to the outgoing radiation from the earth to space, an excellent approximation is to assume that $I\downarrow(0)=0$. The lower boundary, u_1, is customarily taken at the earth's surface. Since all the materials that make up the earth's surface radiate as almost perfect blackbodies beyond 5μ, the upward flux that enters the atmosphere at the lower boundary is nearly the blackbody flux appropriate for the given frequency interval and surface temperature, i.e., $I\uparrow(u_1)=I_b(u_1)$. These boundary conditions will be assumed in the remainder of this article. Equations (6) and (9) can now be written in the following simpler form:

$$I\uparrow(u)=I_b(u)+\int_u^{u_1}\tau(u,v)\frac{dI_b(v)}{dv}dv \qquad (10)$$

and

$$I\downarrow(u)=I_b(u)-I_b(0)\tau(0,u)-\int_0^u\tau(v,u)\frac{dI_b(v)}{dv}dv. \qquad (11)$$

These equations are valid for any arbitrary variation of temperature with height. However, in order to understand the structure of these equations more clearly, let us consider in more detail the solutions for model atmospheres with two particular temperature distributions: (I) temperature constant with height; (II) blackbody intensity varies linearly with u.

For case I, I_b is a constant. Equations (10) and (11) can be written as

$$I\uparrow(u)=I_b, \qquad (12)$$

$$I\downarrow(u)=I_b[1-\tau(0,u)]. \qquad (13)$$

For case II the variation of the blackbody intensity is assumed to be

$$I_b(u)=I_b(0)+au, \qquad (14)$$

where a is a constant. This equation represents the actual temperature variation in the lower atmosphere with fair accuracy. In this case Eqs. (10) and (11) become

$$I\uparrow(u)=I_b(u)+a\int_u^{u_1}\tau(u,v)dv, \qquad (15)$$

$$I\downarrow(u)=I_b(u)-I_b(0)\tau(0,u)-a\int_0^u\tau(v,u)dv. \qquad (16)$$

3. THE RADIATIVE FLUX FOR FOUR MODEL ATMOSPHERES

The particular solutions of the equations of radiative transfer are obtained in this section for four model atmospheres. Those readers who are not interested in details of the mathematical calculation may wish to read only the discussion of the results that is given in Sec. 4.

For each of the four model atmospheres that are discussed in this section, the temperature is assumed to agree with either case I or II of the preceding section. It is also assumed in each case that the fractional concentration of the radiating gas, c, is constant with height. This is the case for a gas such as CO_2 that is uniformly mixed in the atmosphere to great heights, but it would be true only over a small range of heights for a gas such as ozone or water vapor. For the first three model atmospheres it is also assumed that the temperature is a constant independent of height, case I.

a. Absorption Coefficient Independent of Height

The simplest solution to the radiative flux equations is obtained if it is assumed that the absorption coefficient is a constant independent of height. Then it follows immediately from Eqs. (3), (12), and (13) that

$$\begin{aligned}\tau(u_0,u_1)&=\exp[-k(u_1-u_0)\sec\theta],\\ I\uparrow(u)&=I_b,\\ I\downarrow(u)&=I_b[1-\exp(-ku\sec\theta)].\end{aligned} \qquad (17)$$

These equations are valid at a particular frequency if k is a constant independent of height. If an average absorption coefficient can be obtained for a given frequency interval, Eqs. (17) can also be applied to the entire interval. Since the blackbody intensity, I_b, varies relatively slowly with frequency, such average values can be readily computed, if k is also a slowly varying function of the frequency. Unfortunately it is very difficult to compute an appropriate average value for the absorption coefficient because of its very rapid variation with frequency in band spectra.

b. Band of Nonoverlapping Lines with Pressure-Broadened Line Shape

For the range of pressures and temperatures that occur in the earth's atmosphere, the shape of the spectral lines is usually determined by pressure broadening; the absorption coefficient is

$$k(\nu)=(S/\pi)[\alpha/(\nu-\nu_0)^2+\alpha^2], \qquad (18)$$

where S and α are the total intensity and half-width of the spectral line and ν_0 is the frequency of the line center.

At high altitude the Doppler half-width may be as large or larger than the pressure broadened half-width. However, since the Doppler line shape falls off exponentially far from the line center, it can be neglected compared to the pressure-broadened line shape up to at least 50 km.[1] Equation (18) is also not valid for frequencies far from the line center even for pure pressure broadening. Again these deviations occur so far from the line center that they have only a very small influence on the calculation of the atmospheric radiation flux.[2] If the lines in a band are spaced sufficiently far apart so that the transmission is nearly unity at frequencies midway between two adjacent line centers for the path length under consideration, then the total absorption is just the sum of the contributions from each of the nonoverlapping lines in the band.

Both kinetic theory and recent experimental evidence show that the half-width, α, of the spectral lines is proportional to the total pressure,[3,4] so that

$$\alpha=(\alpha_s/p_s)p, \qquad (19)$$

where α_s is the value of the half-width at the standard pressure p_s. With the large pressure range that occurs in the atmosphere, it is very important to take the correspondingly large variation of the half-width with height into account. The half-width also varies in many cases inversely as the square root of the absolute temperature. This variation is only about 20% for the range of temperatures in the atmosphere and is neglected in the examples in this section. However, it is taken into account in the final atmospheric calculations.

For the following calculations a relation is needed between du, dp, and $d\alpha$. Consider a rectangular parallelepiped of unit area and height dz in the atmosphere. The pressure change, dp, between the upper and lower surfaces is balanced by the weight of the parallelepiped. Thus from Eqs. (2) and (19), it follows that

$$dp=-g\rho dz=(g/c)du=(p_s/\alpha_s)d\alpha, \qquad (20)$$

where g is the acceleration of gravity.

It is now possible to evaluate exactly the transmission function for an isothermal atmosphere by the substitution of Eqs. (18), (19), and (20) into Eq. (3) to obtain

$$\tau(u_0,u_1)=\exp\left\{-\frac{Scp_s\sec\theta}{\pi g\alpha_s}\int_{\alpha_0}^{\alpha_1}\frac{\alpha d\alpha}{(\nu-\nu_0)^2+\alpha^2}\right\}, \qquad (21)$$

where α_0 and α_1 are the values of the half-width at the altitudes where the mass of radiating gas per unit area is u_0 and u_1, respectively.

The evaluation of the integral in Eq. (21) gives a logarithm and after some rearrangement the transmission function can be written as

$$\tau(u_0,u_1)=\left[\frac{(\nu-\nu_0)^2+\alpha_0^2}{(\nu-\nu_0)^2+\alpha_1^2}\right]^\gamma, \qquad (22)$$

where $\gamma=Scp_s\sec\theta/2\pi\alpha_s g$. The nondimensional constant γ is the parameter that occurs in all atmospheric radiation problems whenever the variation of the half-width with pressure is taken into account.[5]

Because the transmission varies so rapidly with frequency in a band spectrum, this result for $\tau(u_0,u_1)$ is of little practical interest; it applies only to a particular frequency. Instead the quantity of interest is the radiation flux over some finite frequency interval, $\Delta\nu$. The integrated absorption, $A(u_0,u_1)$ over a finite frequency interval is defined as

$$A(u_0,u_1)=\int_{\Delta\nu}[1-\tau(u_0,u_1)]d\nu. \qquad (23)$$

The total upward radiation flux in this interval is written in boldface type and is defined as

$$\mathbf{I}\uparrow=\int_{\Delta\nu}I\uparrow d\nu \qquad (24)$$

and a similar equation for $\mathbf{I}\downarrow$.

In practice the frequency interval $\Delta\nu$ is always chosen small enough so that I_b does not

[1] G. N. Plass and D. I. Fivel, Astrophys. J. **117**, 225 (1953).
[2] G. N. Plass and D. Warner, J. Meteorol. **9**, 333 (1952).
[3] F. Pedersen, Meteorol. Annaler **1**, 115 (1942).
[4] J. Strong and G. N. Plass, Astrophys. J. **112**, 365 (1950).
[5] G. N. Plass and D. I. Fivel, Quart. J. Roy. Meteorol. Soc. **81**, 48 (1955).

vary appreciably in the interval and at the same time it must be much larger than the half-width of the spectral line. The interval $\Delta\nu$ may be chosen as large as 1μ and may contain hundreds of spectral lines. Thus in integrating Eqs. (8) to (16) over frequency an average value of I_b can be removed from the integral over frequency without appreciable error. For example, after integration over frequency Eqs. (12) and (13) become

$$\mathrm{I}{\uparrow}(u)=I_b\Delta\nu, \tag{25}$$

$$\mathrm{I}{\downarrow}(u)=I_b\Lambda(0,u). \tag{26}$$

If Eq. (22) is substituted in Eq. (23), the integral over frequency can be extended from minus to plus infinity, since the absorption has fallen to zero before another spectral line is reached and since we assumed nonoverlapping spectral lines. Thus the integrated absorption can be written as

$$\Lambda(u_0,u_1)=2\alpha_1\int_0^\infty\left\{1-\left[\frac{x^2+(\alpha_0/\alpha_1)^2}{x^2+1}\right]^\gamma\right\}dx, \tag{27}$$

where $x=(\nu-\nu_0)/\alpha_1$ and where the integral from minus to plus infinity has been written as twice the integral from zero to infinity, since the integrand is an even function of x.

This integral can be evaluated immediately if the parameter γ is a small integer. The first two values are

$$\gamma=1:\quad \Lambda(u_0,u_1)=\pi\alpha_1[1-(\alpha_0/\alpha_1)^2],$$

$$\gamma=2:\quad \Lambda(u_0,u_1)=\tfrac{1}{2}\pi\alpha_1[3-2(\alpha_0/\alpha_1)^2-(\alpha_0/\alpha_1)^4]. \tag{28}$$

Actually the integral of Eq. (27) can be evaluated exactly for any positive, real value of γ in terms of Legendre functions of order γ. The result is[4,6]

$$\Lambda(u_0,u_1)=2\pi\alpha_1\gamma(\alpha_0/\alpha_1)^\gamma[P_\gamma(z)-(\alpha_0/\alpha_1)P_{\gamma-1}(z)], \tag{29}$$

where $z=(\alpha_0^2+\alpha_1^2)/2\alpha_0\alpha_1$. This expression is considerably simplified when $u_0=0$ and from well-known expansions of the Legendre function reduces to[4,6]

$$\Lambda(0,u)=2\pi^{\frac{1}{2}}\alpha\Gamma(\gamma+\tfrac{1}{2})/\Gamma(\gamma), \tag{30}$$

where $\Gamma(x)$ is the gamma function, $\Gamma(x)=\int_0^\infty y^{x-1}e^{-y}dy$.

The upward and downward flux is found from Eqs. (25), (26), and (30) to be

$$\mathrm{I}{\uparrow}(u)=I_b\Delta\nu,$$

$$\mathrm{I}{\downarrow}(u)=I_b\frac{2\pi^{\frac{1}{2}}\Gamma(\gamma+\frac{1}{2})}{\Gamma(\gamma)}\alpha. \tag{31}$$

Thus the intensity of the downward flux at a given height depends on the value of the parameter γ for the particular spectral line. The gamma function can be immediately evaluated if γ is an integer or an integer divided by two.

Since p, u, and α are all proportional to each other from Eq. (20), it follows from Eq. (31) that the downward radiation increases linearly with pressure. The physical reason for this important result is related to the increase in the half-width with pressure. The downward radiation continually increases as the pressure increases since additional radiation is added to the beam from the wings of the broader lines in the lower atmosphere. However, the downward radiation cannot increase indefinitely; eventually the spectral lines begin to overlap and then one of the assumptions made in the derivation of Eq. (31) is no longer valid. However, Eq. (31) always holds from the top of the atmosphere down to some height that depends on the line strength and the average spacing of the lines in the band.[7]

c. Band of Overlapping Lines with Pressure-Broadened Line Shape

For many of the problems connected with atmospheric radiation the overlapping of the various lines in a band must be taken into account. In a typical band the intensities of adjoining lines often vary in an erratic manner and the spacing between adjoining lines varies widely in many spectra. In order to take account of the details of the complicated band structure either a numerical calculation must be made using information about the intensity, frequency and half-width of each spectral line or experimental measurements of the absorption over a range of pressures and path lengths must be

[6] G. N. Plass and D. I. Fivel, J. Meteorol. 12, 191 (1955).

[7] G. N. Plass, J. Meteorol. 11, 163 (1954).

utilized. The latter approach is discussed further in Sec. 5.

However, several simple models have been proposed to represent the essential features of a band spectrum. The Elsasser model of a band is one of the most useful. If a band is assumed to consist only of spectral lines that are all equally intense and are also equally spaced, Elsasser[8] showed that the absorption coefficient for the pressure-broadened line shape is exactly

$$k(\nu)=(S/d)(\sinh\beta/\cosh\beta-\cos\theta), \qquad (32)$$

where $\beta=2\pi\alpha/d$, $\theta=2\pi\nu/d$, and d is the spacing between the lines.

Since the frequency integration is now over the interval d, the integrated absorption $A(u_0,u_1)$ can be written from Eqs. (3) and (23) as

$$A(u_0,u_1)=d-\frac{d}{2\pi}\int_{-\pi}^{\pi}\left[\frac{\cosh\beta_0-\cos\theta}{\cosh\beta_1-\cos\theta}\right]^{\gamma}d\theta, \qquad (33)$$

where β_0 and β_1 are the values of β at u_0 and u_1, respectively. This integral may be evaluated exactly for integral values of γ; the partial derivative of $A(u_0,u_1)$ with respect to either u_0 or u_1 may be evaluated exactly for any positive, real value of γ. The details of these calculations are given by Plass and Fivel.[6]

For our purposes let us merely note that the integral in Eq. (33) can be evaluated immediately if $\gamma=1$. The result is that

$$\gamma=1:\quad A(u_0,u_1)=(d/\sinh\beta_1)(\cosh\beta_1-\cosh\beta_0). \qquad (34)$$

As β approaches zero, the ratio of the spacing to the half-width of the lines becomes larger and the effects caused by overlapping spectral lines disappear. In this limit Eq. (34) reduces to Eq. (28) for nonoverlapping lines. It is possible to estimate the importance of effects caused by the overlapping of the spectral lines from the Elsasser model even though it may apply to an actual band only over a rather narrow frequency interval.

The upward and downward flux obtained from Eq. (34) for an isothermal atmosphere is

$$\gamma=1:\quad \begin{cases} I\uparrow(u)=I_bd \\ I\downarrow(u)=I_b(d/\sin\beta)(\cosh\beta-1), \end{cases} \qquad (35)$$

[8] W. M. Elsasser, Phys. Rev. 54, 126 (1938).

where $\beta=2\pi\alpha/d$. The downward flux initially increases linearly with the pressure as in the case of nonoverlapping lines. However, when the half-width, α, becomes of the same order of magnitude as the spacing between the lines, d, the downward flux increases much less rapidly than before. We now have the physically reasonable result that the downward flux approaches, but never becomes larger than the blackbody flux at large path lengths.

d. Absorption Coefficient Independent of Height; Temperature Varies with Height

As the simplest model atmosphere with a temperature that varies with height, let us assume that the blackbody intensity varies linearly with height, case II, Eq. (14), and that the absorption coefficient is a constant independent of height. Then from Eqs. (15) and (16) it follows that

$$\begin{aligned} I\uparrow(u)&=I_b(0)+au \\ &\quad+(a/k\sec\theta)\{1-\exp[-k(u_1-u)\sec\theta]\}, \\ I\downarrow(u)&=I_b(0)[1-\exp(-ku\sec\theta)] \\ &\quad+au-(a/k\sec\theta)[1-\exp(-ku\sec\theta)]. \end{aligned} \qquad (36)$$

This relatively simple example illustrates many of the features of more complicated problems for nonisothermal atmospheres.

4. DISCUSSION OF RESULTS

Numerical solutions are given in this section for the radiative flux and heating rates of the four model atmospheres discussed in the previous section. The physical factors which influence the results are emphasized here. Although the results in this section are based on equations derived in Sec. 3, the qualitative discussion given here can be followed without reference to the previous mathematical equations.

Four different model atmospheres that correspond to various assumptions about the absorption coefficient and temperature distribution with height are discussed here. In order to avoid continual reference to these assumptions, they will be referred to by letter: (a) the absorption coefficient and temperature are independent of height; (b) a band of nonoverlapping spectral lines with the pressure-broadened line shape, half-width proportional to the pressure, and temperature and fractional concentration of radiating gas independent of height; (c) a band of overlapping spectral lines with the pressure-broadened line shape, half-width proportional to pressure, each line in the band with the same intensity, equal spacing between all lines in the band, and temperature and fractional concentration of radiating gas independent of height; (d) absorption coefficient independent of height, but the temperature varies with height so that the blackbody intensity increases linearly with the mass of radiating gas per unit area, u [Eq. (14)].

In cases (b) and (c) the results are integrated over a finite frequency interval that is large compared to the half-width of the spectral lines, but is still small enough so that the blackbody intensity does not change appreciably over the interval. In cases (a) and (d) the results can either be regarded as referring to the absorption coefficient at a single frequency or to some average absorption coefficient over a finite frequency interval. When there is a line structure to the spectrum, it is very difficult to decide what value should be chosen for an average absorption coefficient. Thus for an actual atmospheric problem cases (b) and (c) are more realistic models than cases (a) and (d).

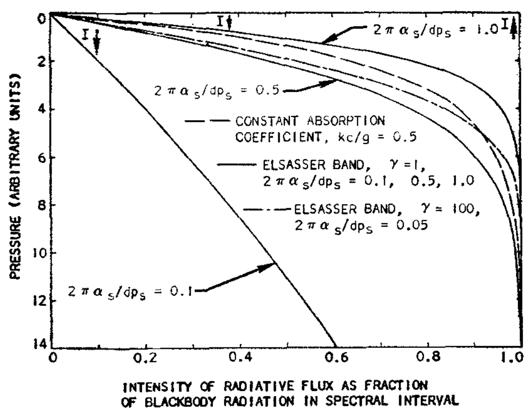

FIG. 1. The upward and downward radiation flux as a fraction of the blackbody radiation in the spectral interval when the temperature does not vary with height. In this case the upward flux is always equal to the blackbody intensity. The downward flux is given for case (a), absorption coefficient independent of height, and for case (c), a band of overlapping spectral lines with the pressure-broadened line shape, half-width proportional to the pressure, each line in the band with the same intensity and equal spacing between the lines (Elsasser band), and the fractional concentration of the radiating gas independent of height. The constant γ is defined below Eq. (22).

In this section all results refer to radiation that is traveling at some particular angle, θ, to the vertical. There is no loss of generality if the reader wishes to consider the radiation as traveling in the vertical direction ($\theta=0$) itself. None of the qualitative results of this section are changed by integration over all of the angles in the hemisphere. Since such an integration usually increases the complexity of the mathematical equations without introducing any essential change in the results, it has been omitted here. In Sec. 5 the methods used for actually performing this integration over the hemisphere are discussed briefly.

The upward and downward radiation flux for cases (a), (b), and (c) are shown in Fig. 1 as a function of the pressure. Since an isothermal atmosphere is assumed in all of these cases with the surface of the earth at the same temperature as the atmosphere, the upward radiation flux is constant at all heights and is equal to the blackbody intensity for this temperature and frequency interval.

Since it was assumed that there is no infrared radiation of appreciable intensity incident from outside the earth's atmosphere, the downward flux is zero at the top of the atmosphere. If the absorption is large in the particular frequency interval, the downward flux approaches the blackbody intensity near the earth's surface. On the other hand, if the absorption is small in the frequency interval, the downward flux rises to only a small fraction of the blackbody intensity at the earth's surface.

The downward flux is given in Fig. 1 for case (a) when $kc/g=0.5$; case (b) when $\gamma=1$, $2\pi\alpha_s/dp_s=0.1$, 0.5 and 1; case (c) when $\gamma=100$, $2\pi\alpha_s/dp_s=0.05$ (α_s and p_s are the half-width and pressure, respectively, at some arbitrarily chosen standard value; d is the spacing between the lines in the band; $\gamma=Scp_s\sec\theta/2\pi\alpha_s g$, a nondimensional parameter that occurs in all radiation problems where the half-width varies with the pressure; S is the intensity of the spectral line). First the influence of the spacing between the lines in a band on the radiative flux is shown by the three curves for $\gamma=1$. Consider the line intensity and half-width to be fixed. Then as the line spacing increases, the parameter $2\pi\alpha_s/dp_s$ decreases. For a relatively small line spacing ($2\pi\alpha_s/dp_s=1$), the downward flux approaches the blackbody intensity only a short distance below the top of the atmosphere. For a larger line spacing ($2\pi\alpha_s/dp_s=0.1$), the downward flux increases with pressure much more slowly. In this case the downward flux increases linearly with pressure over a considerable range of pressures; in this linear region there is no influence from the overlapping of the spectral lines and the results of case (b) are valid.

With very much stronger intensities for the spectral lines ($\gamma=100$), the parameter $2\pi\alpha_s/dp_s$ must be chosen smaller than before in order to obtain a comparable increase in the downward radiation. Thus, as the intensity of the spectral line increases, the mean spacing between the lines must also be increased (roughly as the square root of the intensity) in order to maintain a comparable value for the flux. One such curve is shown in Fig. 1 for comparison.

The downward flux for a constant absorption coefficient, case (a), is also shown in Fig. 1 for the value of the parameter $kc/g=0.5$. This curve has a somewhat different shape than those for the case (c), but has the same qualitative features.

In our actual atmosphere, the temperature varies with height and it is important to consider a simple example of this type. Results for case (d) are shown in Fig. 2. The parameter, ac/g, that determines how rapidly the temperature increases as the pressure increases is chosen to have the value $\frac{1}{3}$. The blackbody flux appropriate to the assumed temperature at each height is shown as the dashed straight line in Fig. 2. The upward and downward flux is drawn for $kc/g=5$, 0.5, and 0.05.

For strong absorption ($kc/g=5$) the upward flux is slightly greater than the blackbody flux at a given height in the atmosphere, except at the surface of the earth (here taken at $p=14$ on an arbitrary scale) where they must be equal. As the line strength decreases the upward flux becomes larger at a given height in the atmosphere. For a weak line ($kc/g=0.05$) the upward flux decreases only very slightly from its value at the surface of the earth. For such a weak line there is not sufficient interaction between the

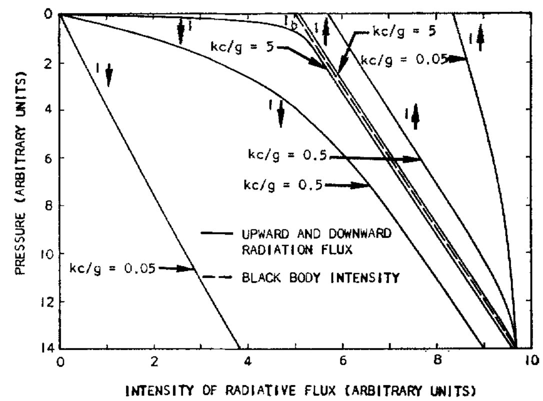

FIG. 2. The upward and downward radiation flux when the temperature varies with height. It is assumed that the blackbody intensity increases linearly with the mass of radiating gas per unit area, case (d), Eq. (14). The absorption coefficient is assumed to be independent of height. Curves are shown when $ac/g=\frac{1}{3}$ and $kc/g=0.05$; 0.5; 5. For comparison the blackbody flux appropriate to the temperature at the given height is shown as the dashed curve.

initial upward flux and the absorbing molecules at a given height to materially reduce the upward flux below its initial value.

Similar considerations determine the downward flux which is assumed to be zero at the top of the atmosphere. For strong absorption ($kc/g=5$) the downward flux increases very rapidly to a value only slightly less than the blackbody radiation at that height. It is important to note that the upward flux is always slightly greater than the blackbody flux for any finite absorption coefficient; the downward flux is correspondingly less than the blackbody flux by a small amount. As the line strength decreases, the downward flux decreases at a given height in the atmosphere. For a weak line the downward flux has still a relatively small value even at the surface of the earth.

Arbitrary temperature distributions in the atmosphere have been chosen for these examples. Since these do not correspond to equilibrium distributions, the radiation either heats or cools each layer through which it passes. If only radiative processes acted to control the temperature in the atmosphere, then the original temperature distribution would change until the equilibrium radiative temperature distribution was reached. In the actual atmosphere other types of processes do not allow this to occur, at least over the entire atmosphere.

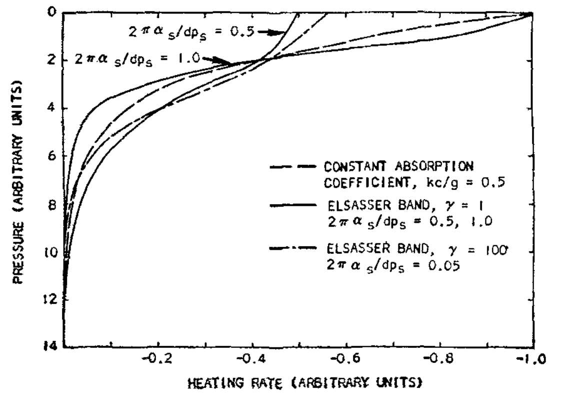

FIG. 3. The heating rate of the atmospheric layers from the infrared radiation for cases (a) and (c). Since the temperature is assumed constant throughout the atmosphere, the radiation always acts to cool the atmospheric layers (negative heating rate).

From the definition of specific heat at constant pressure and from Eq. (2), it follows that the change in temperature with time at a given height in the atmosphere is related to the derivative of the difference of the upward and downward radiation flux by

$$\frac{\partial T}{\partial t}=\frac{c\Delta\nu}{c_p}\frac{d}{du}(I\uparrow-I\downarrow), \qquad (37)$$

where c_p is the specific heat at constant pressure, c is the fractional concentration of the radiating gas, and $\Delta\nu$ is the frequency interval considered.

In many atmospheric problems the heating or cooling rate due to the radiation flux is of considerable interest. It can readily be computed for our four model atmospheres. For an initial isothermal temperature distribution, a radiating gas with a fractional concentration independent of height always acts to cool the atmosphere. The temperature would soon change if no other factors acted to maintain the original temperature distribution. Some results are given in Fig. 3 for case (a) when $kc/g=0.5$ and for case (c) when $\gamma=1$, $2\pi\alpha_s/dp_s=0.5$; $\gamma=1$, $2\pi\alpha_s/dp_s=1$; and $\gamma=100$, $2\pi\alpha_s/dp_s=0.05$. In all of these cases the cooling is greatest at the top of the atmosphere and then falls off rapidly as the pressure increases. The exact shape of the curves depends on the parameters of the relevant band. Even though band absorption for some values of the parameters happens to give less cooling at the top of the atmosphere than is obtained with continuous absorption, it is seen that the band absorption can give considerably greater cooling at lower levels in the atmosphere.

The cooling rate is given in Fig. 4 for case (d) when the temperature decreases with height. Again the infrared radiation always acts to cool the atmosphere provided the fractional concentration of the radiating gas is constant with height. The larger absorption coefficients give the greater cooling rates. These are large not only at the top of the atmosphere, but also at the surface of the earth. The reason for the large cooling rate at the surface of the earth is that a temperature discontinuity always develops at a lower boundary between an atmosphere with a finite absorption coefficient in radiative equilibrium and a black surface. In the actual atmosphere such a discontinuity causes turbulence near the ground which acts to smooth out the temperature distribution and remove the discontinuity.

5. CALCULATION OF INFRARED FLUX FOR THE EARTH'S ATMOSPHERE

A method for the calculation of the radiative flux for actual atmospheric conditions is discussed in this section and the results are presented in the following sections. There are other possible methods that may be used in order to obtain

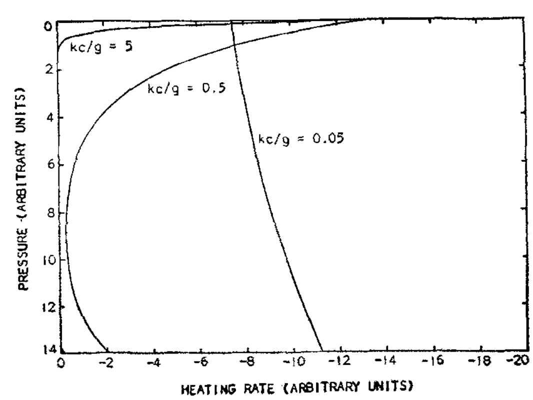

FIG. 4. The heating rate of the atmospheric layers from infrared radiation for case (d). The original temperature distribution decreases with height and the radiation acts to cool the atmospheric layers. The scale for the heating rate is different for each of the three curves. If the arbitrary scale at the bottom of the figure is used for the curve with $kc/g=0.05$, then the heating rates for the curves with $kc/g=0.5$ and 5 should be multiplied by 5 and 50, respectively.

the atmospheric radiation flux, but extensive results have been calculated so far only by the method that is described here. Because these calculations are considerably more complicated than the examples given in Sec. 4, only a qualitative description of the method is given. It is valid for an arbitrary dependence of the temperature on height and takes into account the variation of the spectral line intensity and half-width with temperature and pressure. The calculations are considerably simplified if accurate infrared absorption measurements are available over a range of pressures and path lengths. The infrared flux in the atmosphere can then be calculated from the results of such measurements.[9]

Accurate measurements of the infrared absorption over long path lengths and a considerable range of pressures are now available for the 9.6μ band of ozone[10] and the 15μ band of carbon dioxide.[11] The only other atmospheric gas which appreciably influences the infrared flux is water vapor. Unfortunately absorption measurements of the scope necessary for atmospheric radiation calculations are not yet available for water vapor.

The absorption measurements of Cloud for carbon dioxide are shown in Fig. 5 where the logarithm of the fractional absorption (unity minus the transmission) is plotted against the logarithm of pw, where p is the pressure and w is the density of the radiating gas times the path length used in the laboratory. The corresponding data for ozone from Summerfield's experiments are shown in Fig. 6. It is remarkable that all of these measured values fall on a single universal curve even though the path length and pressure were varied independently in these experiments over a very wide range.

Whenever such a universal absorption curve is observed experimentally for a certain range of pressures and path lengths, there is a simple physical interpretation of the results. Over this range the absorption of radiation is complete at frequencies within several half-widths of the centers of the stronger spectral lines in the measured frequency interval. Whenever the experimental results lie on the universal curve within the experimental error, then it can be shown[9] that the transmission function that is applicable to the variable conditions in the atmosphere can be obtained from the measured absorption curve merely by replacing the variable pw by $g^{-1}\sec\theta\int_{p_0}^{p_1} cp\,dp$, where p_0 and p_1 are the pressures at the two levels between which one wishes to calculate the transmission.

This substitution of variable gives a result for the transmission along the variable atmospheric path that is as accurate as the original laboratory measurements. This substitution is valid regardless of how irregularly the spacing, intensity or

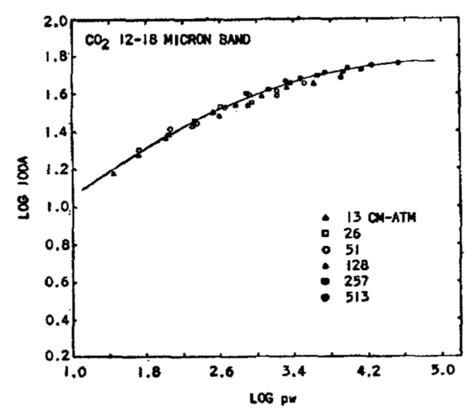

FIG. 5. The fractional absorption, A, of carbon dioxide as a function of the product of the pressure, p, and the optical path length used in the laboratory, w, for the spectral interval from 12–18μ. The pressure is measured in cm-Hg and the path length in cm-atmos. The measurements were made by Cloud.[11]

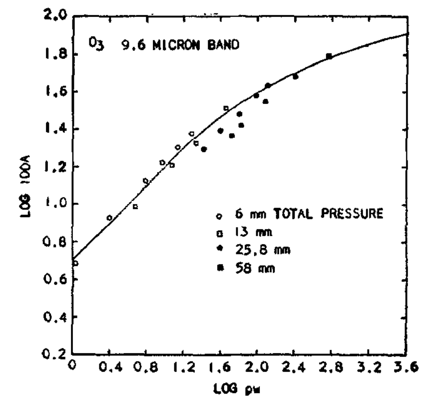

FIG. 6. The fractional absorption, A, of ozone as a function of the product of the pressure, p, and the optical path length used in the laboratory, w. The pressure is measured in mm-Hg and the path length in mm-atmos. The measurements were made by Summerfield.[10]

[9] G. N. Plass, J. Opt. Soc. Am. 42, 677 (1952).
[10] M. Summerfield, The effect of pressure on the infrared absorption of ozone, thesis (Pasadena, 1941, unpublished).
[11] W. H. Cloud, *The 15 Micron Band of CO_2 Broadened by Nitrogen and Helium* (Johns Hopkins University, Baltimore, 1952).

half-width of the spectral lines vary throughout the band and of whether or not the lines overlap, provided only that the range of pressure and path length is in the experimentally determined region for the validity of the universal curve. This result is also valid for any line shape that has an absorption coefficient proportional to the pressure in the wings of the lines, as is true for the Lorentz line shape or any other collision-broadened line shape that has been proposed.

When the logarithm of the absorption is plotted against the logarithm of pw, as in Figs. 4 and 5, the slope of the curve is zero when there is virtually complete absorption. As the absorption decreases, the slope increases to the value one-half when the square-root absorption region is reached. In between these regions the slope gives a convenient measure of the degree to which the spectral lines overlap. If still smaller values of the absorption were measured, the slope would increase to unity in the linear absorption region. In this region, there would be no single absorption curve valid over a wide range of the variable p and w.

It should be emphasized that the universal absorption curve is valid only for long path lengths and for pressures less than a certain critical value. If these variables exceed the experimentally determined limits, then the absorption is a function of both the pressure and path length separately. Fortunately, for most of the range of path lengths that are of interest in the atmosphere, the universal absorption curve is valid. However, in some cases, such as for very short path lengths, the results have to be obtained by extrapolation of the experimental curves.[7,9] Although there is a larger uncertainty associated with transmission functions that are obtained in this manner, it can be shown that this can have an appreciable influence on the final results only at the highest altitudes.

In the actual calculation of the transmission function some further corrections have to be introduced. At the highest altitudes considered (up to 75 km), it was necessary to consider the importance of Doppler broadening to the line shape as well as pressure broadening, but it is not difficult to make this correction.[1]

Unfortunately no suitable laboratory absorption measurements in the infrared have been made as yet over a range of pressures and path lengths at temperatures below room temperature. Since both the intensity and half-width of a spectral line vary with temperature, it is important to take these variations into account for a spectrum such as the 15μ band of carbon dioxide. The procedure for making this correction is fairly complicated[12]; however, it can be shown that even this correction does not appreciably change the accuracy of the final results.

All of the preceding remarks have referred to radiation that is traveling at a certain angle θ to the vertical. However, the final quantity of interest is the diffuse radiation, defined as the beam radiation from all angles integrated over a hemisphere. In many calculations it has been customary to take this approximately into account by multiplying the path length by some constant factor near 1.67. Because this factor does vary between one and two as the transmission in the spectral interval changes from one to zero, certain errors can be introduced in radiation calculations by using such a constant factor.[13] In the atmospheric calculations discussed in the following sections, the path length is multiplied by a variable numerical factor that can be calculated from the transmission function between the levels in question. This procedure gives a considerably more accurate result for the beam radiation than is obtained by the use of a constant factor.

In order to calculate the infrared radiation flux and heating and cooling rates from Eqs. (6), (9), and (37), the transmission function between the two altitudes, z_0 and z_1, $\tau(z_0,z_1)$, was calculated for every pair of heights such that $z_0=0, 1, 2, \cdots,$ 75 km and $z_1=0, 1, 2, \cdots,$ 75 km. The complete procedure for the calculation of these quantities was coded for the MIDAC high-speed digital computer and the results could be obtained for a number of different frequency intervals as well as for various concentrations and distributions of the radiating gases.

6. THE 15-MICRON BAND OF CARBON DIOXIDE

An excellent survey of previous work on carbon dioxide absorption has been given by

[12] G. N. Plass, Quart. J. Roy. Meteorol. Soc. (to be published).
[13] G. N. Plass, J. Meteorol. 9, 429 (1952).

Elsasser and King.[14] The pioneer work of Ladenburg and Reiche[15] has been extended in recent years by Callendar,[16] Elsasser,[17] Kaplan,[18] and Yamamoto.[19] Kaplan[20] and Elsasser and King[14] have made extensive calculations of the absorption from the carbon dioxide band.

In this section the results of the calculations of Plass[12] are described. All of the physical factors that are known to influence this problem have been taken into account. These results are an excellent illustration of the type of solution that can be obtained for the equations of radiative transfer for complicated boundary conditions. The temperature distribution that was assumed for these calculations is given in Fig. 7, curve D. This curve agrees closely with the average temperature distribution at middle latitudes. The spectral region from 12–18μ that is appreciably influenced by the carbon dioxide band was divided into six intervals, each one micron wide, for the purposes of this calculation.

The upward and downward flux is given in Fig. 8 for three different carbon dioxide concentrations in the atmosphere. The present value for the fractional concentration of carbon dioxide is 5×10^{-4}. The upward and downward radiation flux is given for the entire spectral interval from 12–18μ and for three smaller intervals. In order to present as many of the results as possible in a reasonable space, the results for three pairs of intervals have been combined. Thus, instead of showing the flux for the 12–13μ and 17–18μ intervals separately as calculated, only the total is given for both intervals; the variation with height of the results for these two intervals is almost the same and no essential information is lost by combining the results in this manner. For the same reason the results for the 13–14μ and 16–17μ intervals were combined as were those for the 14–15μ and 15–16μ intervals.

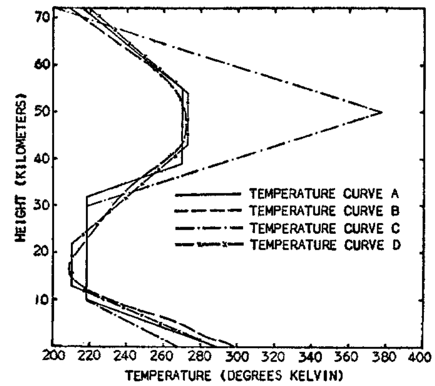

FIG. 7. Various temperature distributions. Temperature curves A, B, and C were used for the calculations of the effect of ozone; temperature curve D was used for carbon dioxide.

These three pairs of intervals show the typical variation of the radiation flux for spectral regions that are nearly transparent, nearly completely absorbing and with an intermediate value for the transmission function. First let us consider the results calculated for the present carbon dioxide concentration. As discussed in Sec. 4 the change in upward flux with height depends on the transmission of the spectral region in question. If the transmission were unity in a given spectral interval, the higher atmospheric layers which have the lower temperatures would be unable to change the upward flux and it would be constant with height. If the transmission were zero, the upward flux would be equal to the blackbody intensity at every height and for our assumed temperature distribution would have a minimum from 13 to 22 km and a maxima at the ground and from 43 to 54 km.

Various intervals in Fig. 8 are close to these limits. For the 12–13; 17–18μ interval the transmission is near unity; the upward flux is nearly constant with a value above 13 km that is only 8% lower than the value at the ground. For the 13–14, 16–17μ interval the transmission has an intermediate value at the lower altitudes; the upward flux falls to almost one-half of its value at the ground between 13 and 22 km. It increases slightly above 22 km, but this rise is very small, since the transmission is so near unity for paths at the higher altitudes that higher temperatures there can have little effect. For the 14–16μ interval the transmission is nearly zero up to 15

[14] W. M. Elasser and J. I. King, *Stratospheric Radiation Technical Report No. 9* (University of Utah, Salt Lake City, 1953).
[15] R. Ladenburg and F. Reiche, Ann. Physik **42**, 181 (1911).
[16] G. S. Callendar, Quart. J. Roy. Meteorol. Soc. **67**, 263 (1941).
[17] W. M. Elasser, *Heat Transfer by Infrared Radiation in the Atmosphere*, Harvard Meteorol. Studies, No. 6 (1942).
[18] L. D. Kaplan, J. Chem. Phys. **18**, 186 (1950).
[19] G. Yamamoto, Science Reports Tohoku Univ., Series 5; Geophysics **4**, 9 (1952).
[20] L. D. Kaplan, J. Meteorol. **9**, 1 (1952).

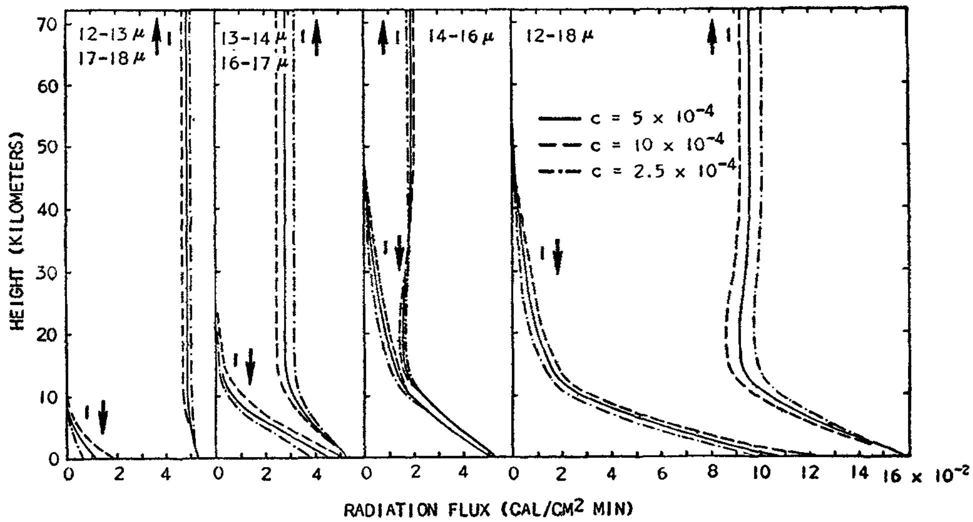

FIG. 8. The upward and downward radiation flux as a function of height for the combined frequency intervals from 12–13 and 17–18μ, from 13–14 and 16–17μ, from 14–16μ, and for the entire interval from 12–18μ. Curves are given for the following carbon dioxide concentrations: $c=5\times10^{-4}$ (0.033% by volume); $c=10\times10^{-4}$ (0.066% by volume); $c=2.5\times10^{-4}$ (0.0165% by volume). The temperature curve D (Fig. 7) was used for these calculations.

km; the upward flux is very close to the blackbody curve for the assumed atmospheric temperature distribution up to 22 km. Above this height the upward flux increases to a value that is 28% above its minimum value; the transmission is still small enough to allow the upward flux to increase to some extent as the atmospheric temperature increases at these heights. The upward flux for the entire range from 12–18μ exhibits the characteristics of a curve for a transmission function with a value intermediate between zero and unity.

Somewhat similar remarks apply to the downward radiation flux. In this case, if the transmission function were unity in a given frequency interval, there would be no downward flux, since the atmosphere could not radiate at these frequencies. If the transmission function were zero, the downward flux would equal the blackbody intensity for the appropriate temperature at a given height. For the interval from 12–13, 17–18μ the downward flux is quite small and has an appreciable value only below 10 km. For the interval from 13–14, 16–17μ the downward flux has a considerably larger value. For the interval from 14–16μ, the downward flux is larger than for either of the previous intervals and approaches the blackbody curve below 13 km. The upward and downward flux would have an identical value equal to the blackbody intensity below 13 km for the 14–16μ interval, if the transmission function were actually zero for all possible path lengths below 13 km. Actually for sufficiently small path lengths, even near the surface of the earth, the transmission function approaches unity; this is the reason for the small difference between the upward and downward flux in this case (compare with Fig. 2).

The cooling rate due to the entire carbon dioxide band from 12–18μ is shown in Fig. 9. From the ground up to 10 km the cooling rate is of the order of a few tenths of a degree Centigrade per day. The rate rises rapidly from 22 to 43 km and reaches a maximum of 5.8°C per day at 43 km. The cooling rate is above 4.0°C per day from 38 to 55 km. The rate decreases rapidly at higher altitudes. The reason for the maxima and minima in this curve at 13, 22, 43, and 54 km arises from the discontinuities in the derivative of the assumed temperature curve at these altitudes; a further discussion is given in Sec. 7. Such discontinuities seldom, if ever, actually exist in the atmosphere. Therefore the values in the neighborhood of these maxima and minima should be smoothed out slightly when comparisons are made with the actual atmosphere.

The effect of changes in the carbon dioxide concentration is shown in Figs. 8 and 9. Curves are given for carbon dioxide concentrations of 0.025, 0.05, and 0.10% by weight or 0.0165, 0.033, and 0.066% by volume. The lowest and highest values correspond to halving or doubling the present atmospheric carbon dioxide amount.

The variation of the upward and downward flux can be interpreted in terms of changes in the transmission function. An increase in the carbon dioxide concentration decreases the transmission function. Thus for the intervals from 12–13, 17–18μ and from 13–14, 16–17μ the upward flux is smaller the larger the carbon dioxide concentration, since the small transmission function allows the flux to approach closer to the black-body flux at the temperature minimum from 13 to 22 km. The same is true for the interval from 14–16μ below 32 km. Above this height the upward flux is slightly greater for larger carbon dioxide concentrations since the transmission function is still sufficiently small in this region so that the upward radiation can increase slightly due to the higher temperatures in the upper part of the stratosphere.

Similar remarks apply to the downward flux. In this case the larger carbon dioxide concentrations give the larger values for the downward flux. If the carbon dioxide concentration doubles, the total radiation flux leaving the earth to space in the interval from 12–18μ increases about 5%; the downward flux received at the surface of the earth increases by slightly more than 10%.

The cooling rates are given in Fig. 9 for these three different carbon dioxide concentrations. There is no definite difference in the cooling rates within the accuracy of the calculation below 24 km and only one curve is shown in this region. At higher altitudes the cooling rate increases appreciably if the carbon dioxide concentration changes. At the maximum of the cooling curve at 43 km, the cooling rate increases from 5.8°C per day to 8.5°C per day if the carbon dioxide concentration is doubled.

From the change in the radiation flux at a given height with variations in the carbon dioxide concentration it is possible to estimate the change in the equilibrium temperature at the surface of the earth. It is assumed that nothing else changes that can alter the radiation balance when the carbon dioxide amount varies. In order to obtain the temperature change it is also assumed that an additional amount of radiant energy equal to 0.0033 cal/cm^{2}min would be radiated to space, if the average temperature of the earth's surface increases 1°C. This number cannot be calculated accurately until a detailed study has been made of the water vapor spectrum, but it represents the best value that can be given with our present knowledge of this spectrum. When a more accurate value for this number is obtained in the future, all of the temperature changes given here should be multiplied by the ratio of the new to the old value.

At the surface of the earth the change in the downward flux is 0.0119 and 0.0125 cal/cm^{2}sec when the carbon dioxide amount is doubled and halved, respectively. Thus, in order to restore equilibrium the surface temperature must rise 3.6°C if the carbon dioxide concentration is doubled and the surface temperature must fall 3.8°C if the carbon dioxide concentration is halved. It is interesting to note that the four intervals from 12–13, 13–14, 16–17, and 17–18μ contribute about equally to this temperature change. The interval from 14–16μ does not contribute since it is opaque near the ground at all of these concentrations. The argument has sometimes been advanced that the carbon

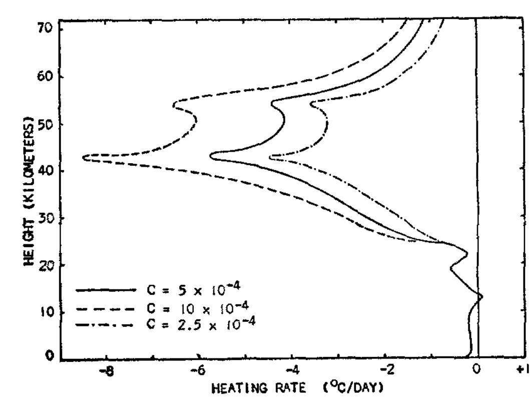

FIG. 9. The cooling rate in °C/day for the entire frequency interval from 12–18μ. Curves are given for the following carbon dioxide concentrations: $c=5\times10^{-4}$ (0.033% by volume); $c=10\times10^{-4}$ (0.066% by volume); $c=2.5\times10^{-4}$ (0.0165% by volume). The temperature curve D was used for these calculations. There was no difference in the cooling rates below 24 km within the accuracy of the calculation and only one curve is shown in this region.

dioxide cannot cause a temperature change at the surface of the earth because the carbon dioxide band is always black at any reasonable concentration. This argument is true for the lines near the center of the band from 14–16μ, but neglects completely the important contribution of the lines farther from the band center.

By this same method it is found that the average temperature is lowered 2.2°C and 1.3°C at the upper surface of a cloud at 4 km and 9 km, respectively, if the amount of carbon dioxide in the atmosphere is halved. This temperature change at the upper surface of a cloud could be the cause of the increased precipitation at the beginning of a glacial period. The current status of the carbon dioxide theory of glaciation is discussed by Plass in another article.[21]

7. THE 9.6-MICRON BAND OF OZONE

As a final example of the solutions for the equations of radiative transfer, the infrared flux in the frequency interval near the 9.6μ band of ozone is discussed in this section. The many aspects of the ozone problem have been discussed in the excellent survey articles by Craig[22]; these contain references to earlier work. Especial mention should be made of the long study of ozone cooling rates by Gowan, of which the most recent was published in 1947.[23] Recent articles by Wexler,[24] Craig,[22] Johnson,[25] and Pressman[26] have made interesting contributions to the ozone problem and have summarized more recent work.

The same method that is described in Sec. 6 has been used[27] to calculate the influence of various ozone and temperature distributions on the infrared flux. The absorption measurements of Summerfield[10] shown in Fig. 6 are the basis of these calculations.

The curves L, M, and N of Fig. 10 show the three different ozone distributions that were assumed for these calculations. Each of these curves corresponds closely to the actual measurements of ozone concentration that have been made from rockets. Curve L represents the average ozone concentration at middle latitudes as well as it is known at the present time. Curve M has higher than normal ozone amounts at high altitudes and the maximum of the ozone curve occurs at very low altitudes. Vertical convection currents can carry the ozone down from higher altitudes, thus causing the maximum to occur at low altitudes. Curve N represents a condition when considerably less than the normal ozone amount is found above 30 km.

The total ozone amounts for curves L, M, and N are 0.213, 0.267, and 0.254 cm, respectively, at standard temperature and pressure. The curves in Fig. 10 have been plotted against the logarithm of the concentration as abscissa and not against the concentration directly as is customary; thus they have a somewhat different shape than usual. Curves M and N were chosen to represent the more or less extreme variations that might be found in the atmosphere from an average ozone amount represented by curve L.

Temperature curve A in Fig. 7 represents with reasonable accuracy the average temperature distribution at middle latitudes. The differences between this curve and curve D that was used for the carbon dioxide calculations is small and has no significance except that these calculations were made at different times.

Temperature curve B agrees with the values

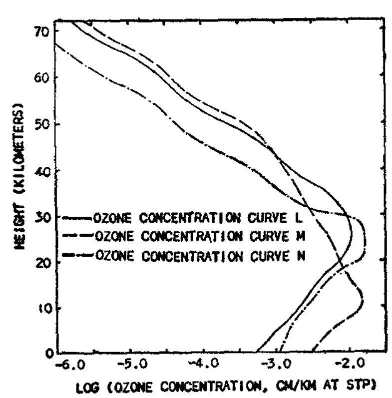

FIG. 10. Assumed variation with height of the ozone concentration (curves L, M, and N). The ozone concentration is expressed in cm/km at STP.

[21] G. N. Plass, Am. J. Phys. 24, 376 (1956).
[22] R. A. Craig, Meteorol. Monographs 1, No. 2 (1950); *Compendium of Meteorology* (American Meteorological Society, Boston), p. 292.
[23] E. H. Gowan, Proc. Roy. Soc. (London) **A190**, 219, 227 (1947).
[24] H. Wexler, Tellus 2, 262 (1950).
[25] F. S. Johnson, Bull. Am. Meteorol. Soc. **34**, 106 (1953); Proc. Toronto Meteorol. Conf. 1953, p. 17 (1954).
[26] J. Pressman, J. Meteorol. 12, 87 (1955).
[27] G. N. Plass, Quart. J. Roy. Meteorol. Soc. **82**, 30 (1956).

adopted by the Rocket Panel[28] as the best average temperatures that can be given at the present time. The calculations were repeated using curve B in order to study the effects of the discontinuities of slope in curve A on the cooling rate.

Temperature curve C was arbitrarily drawn to study the effect of a sudden heating in the upper atmosphere (to a maximum of 378°K at 50 km) and of somewhat cooler temperatures near the surface of the earth than are given by curve A. All of the densities used in this calculation are those given by the Rocket Panel[28] corrected where necessary for the assumed variation of the temperature.

The upward and downward infrared flux in the region of the 9.6μ ozone band is shown in Fig. 11 for ozone distribution curves L, M, and N and temperature curve A. All values of the flux have been reduced to an interval 1.07 microns wide centered at 9.6μ. Curve M has the highest ozone concentration at low altitudes and therefore the upward flux is more nearly able to follow the decrease in the blackbody intensity with height up to 10 km. For this reason the values of the upward flux are less for curve M at all heights than for N and L. The transmission is so near unity above 30 km that the temperature rise in this region causes only a small increase in the upward flux. The variations in the downward flux shown in Fig. 11 can be explained by similar considerations.

The rate of heating or cooling due to the absorption and emission of infrared radiation by the ozone is plotted in Fig. 12. Any gas must cool the atmosphere by the action of infrared radiation in the layer next to the earth's surface, if it is assumed that the atmospheric boundary layer is at the same temperature as the earth's surface. This layer of cooling is found to exist for each ozone distribution curve in Fig. 12. If the mixing ratio is constant or decreases with height (as is usually the case for carbon dioxide and water vapor), the action of the infrared radiation can only cool the atmosphere at all levels. However, if the mixing ratio increases with height (as is the case with ozone up to heights of 10 to 30 km), then the infrared radiation can cause a net heating of the atmosphere over a range of heights. Figure 12 shows that this effect is most pronounced at moderate altitudes for curve M which has the highest ozone concentration at low altitudes. The ozone distribution M causes cooling above 13 km, whereas ozone distributions L and N continue to warm the atmosphere to heights of 20 km and above. The reason for this difference is the greater altitude at which the maximum of the ozone distribution occurs for curves L and N than for M. The heating rates are quite small, never exceeding 0.2°C/day at these altitudes.

The cooling rates begin to become large around 35 km for all three curves and are appreciable up to 60 km. The maximum cooling shown for any of these curves is 2.7°C/day. The cooling rate for curve N is roughly one-half that of the other curves above 40 km, because

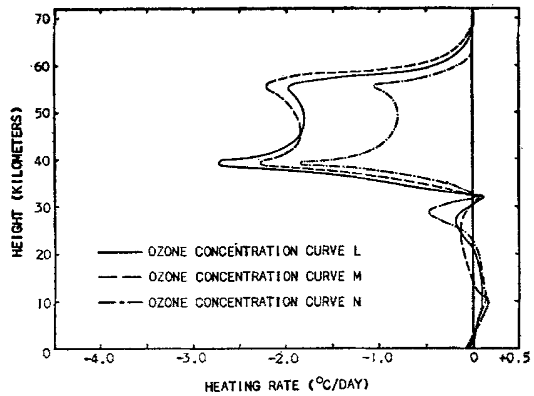

FIG. 12. The heating rates for ozone distribution curves L, M, and N and temperature curve A.

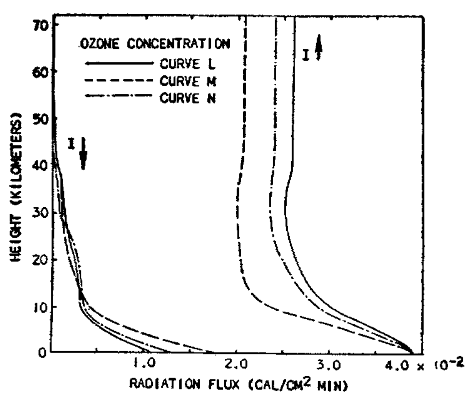

FIG. 11. The upward and downward radiation flux in the region of the 9.6μ ozone band for ozone distribution curves L, M, and N and temperature curve A.

[28] Rocket Panel, Phys. Rev. 88, 1027 (1952).

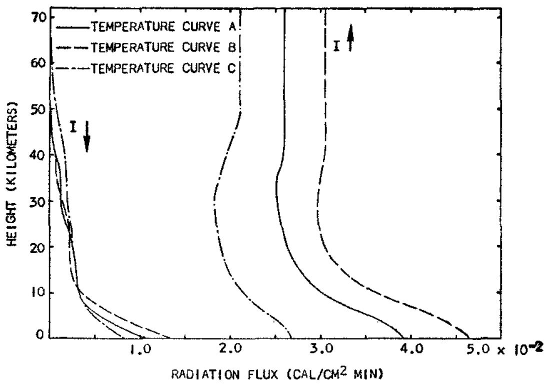

FIG. 13. The upward and downward radiation flux in the region of the 9.6μ ozone band for ozone distribution curve L and temperature curves A, B, and C.

of the very small ozone concentration assumed at these altitudes for this curve. The double maximum is caused by the discontinuities in the assumed temperature curve and is discussed below.

The normal variations in ozone amount in the atmosphere at a given altitude are probably represented by the three rather different values for the ozone concentrations given by curves L, M, and N at each altitude. Thus the results of Figs. 11 and 12 show the approximate variation in infrared flux and cooling rate that might be expected to occur in the atmosphere due to reasonable variations in the ozone amount. It appears that variations in the ozone amount can change the cooling rate by as much as a factor of two. However, if the temperature remains unchanged, the qualitative features of the cooling curve including the position of the maxima do not vary appreciably with changes in the ozone concentration.

In order to separate the effects due to temperature variation with height from those due to changes in the ozone distribution curve, the calculations were repeated using the single ozone distribution curve L, with the three different temperature curves A, B, and C (Fig. 7).

The corresponding upward and downward flux of radiation is shown in Fig. 13. Since the temperature in the atmospheric layers close to the ground is the principal factor that determines the magnitude of the upward flux for a given ozone distribution, it follows that at all heights the upward flux corresponding to the temperature curve C is less than that from curve A, which in turn is less than that from curve B. The assumed ground temperatures increase in the same sequence. The upward flux corresponding to curve C continues to increase at higher altitudes (up to 50 km) than for the other curves due to the very high assumed temperature at 50 km for curve C. Similarly the downward flux is considerably greater for curve C than for the other two at all altitudes above 30 km. Near the surface of the earth, the downward flux is mainly determined by the temperature of the lowest atmospheric layers. Below 7 km the three values for the downward flux are in the same numerical sequence as the corresponding assumed temperatures.

In Fig. 14 the heating and cooling rates are shown for these three temperature curves. The region of cooling is seen to extend from the earth's surface to a few kilometers with a region of heating from there to above 20 km. The magnitude of the heating or cooling is again less than a few tenths of a degree per day in this region.

The effects introduced by a temperature curve composed of regions with a linear variation of temperature with height and discontinuities in the derivative (curve A) compared to a temperature distribution with a continuous derivative (curve B) are shown in Fig. 14. The points at which the derivative of the temperature with respect to height are discontinuous appear in Fig. 14, curve A, as maxima in the cooling curve

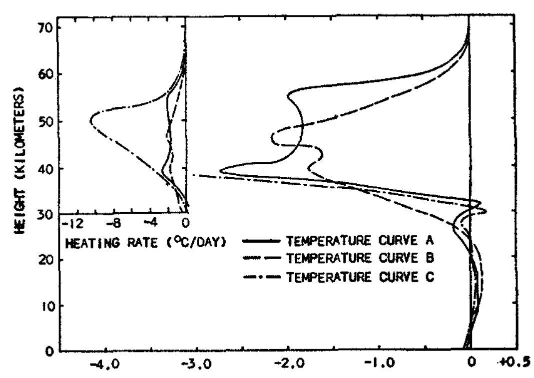

FIG. 14. The heating rates for ozone distribution curve L and temperature curves A, B, and C. The insert in the upper left part of the figure shows the heating rate in °C/day at the higher altitudes plotted to a different scale.

at 39 and 55 km and a narrow maximum of heating at 32 km.

The high assumed temperatures near 50 km for curve C give such a high cooling rate above 40 km that it has been plotted on another scale as an insert to Fig. 14. There is a narrow region of heating near 30 km and a maximum cooling rate at 50 km; these are the positions of the discontinuities in temperature curve C. The cooling rate is 10.6°C/day at 50 km where the assumed temperature is 105°C compared to a maximum of 2.7°C/day for curve A at 39 km where the assumed temperature is −3°C. Above 30 km the results of Fig. 14 agree qualitatively with the earlier calculations of Gowan[23] (see also the discussion by Wexler[24]).

Thus if some process, such as increased ultraviolet radiation from the sun, increases the temperature of the upper layers of the atmosphere, the infrared radiation immediately acts to cool these layers. It is interesting to compare the separate cooling rates due to ozone and carbon dioxide as given in Table I. The ozone cooling rates are for the ozone concentration curve L and the temperature curve B. A small correction has been made to the carbon dioxide results between 40 and 60 km given in Table I in order to bring them more closely into agreement with the results that would have been obtained had the temperature curve B been used. The largest relative contribution from the ozone is at 45 km where it contributes 30% to the total energy radiated from both ozone and carbon dioxide. The carbon dioxide is much more important than the ozone in cooling the atmosphere above 55 km.

TABLE I. Cooling rates in the stratosphere.

Height	Ozone	Carbon dioxide	Total
25 km	0.1°C/day	1.3°C/day	1.4°C/day
30	0.5	2.4	2.9
35	1.3	3.3	4.6
40	1.8	4.5	6.3
45	2.1	4.9	7.0
50	1.5	4.6	6.1
55	0.8	4.0	4.8
60	0.3	2.3	2.6
65	0.1	1.7	1.8
70	0.0	1.3	1.3

It is interesting to compare these cooling rates with the absorption of solar energy by ozone at these altitudes as calculated by Johnson[25] and Pressman.[26] Since the heating rates vary greatly with altitude and season, it is difficult to compare these results quantitatively. However, it is clear that the heating and cooling rates agree qualitatively and have their maxima at the same altitude (45 km) with approximately the same value of 7.0°C/day. The cooling due to water vapor is not included in Table I. Differences between the heating and cooling rates could be ascribed to the possible additional cooling effect of water vapor and to uncertainties in the calculations themselves.

These results show that the calculated absorption of ultraviolet and visible radiation in the stratosphere is approximately equal to the cooling effect of the infrared radiation. Unlike the troposphere where many different processes may change the temperature of a parcel of air, the average temperature distribution in the stratosphere appears to be determined primarily by the absorption and emission of radiation.

Summer Institute for Physics Teachers

A Summer Institute of Physics, for teachers of physics in both college and high schools, will be held at the University of Wyoming, July 16–August 17, 1956, under the sponsorship of the National Science Foundation. This Institute will emphasize recent developments in physics and their implications for the teaching of both high school and general college and university physics.

The Wyoming Institute will also include consideration of the better integration of courses in high school and college physics and procedures for identifying and attracting gifted students into careers in physics. The high school group will be under the direction of Dr. S. Winston Cram, head of the Physical Sciences Department of the Kansas State Teachers College, Emporia, Kansas. Professor Cram is the Chairman of the AAPT Committee on Secondary School Physics. The college group will be led by Dr. Hugh C. Wolfe, head of the physics department at the Cooper Union, New York City. A series of lectures by prominent visiting physicists will also feature the morning sessions.

Stipends of $250 plus an allowance of $65 per dependent are available for enrollees. The director of the Institute is Professor Marsh W. White of The Pennsylvania State University. A leaflet describing this program and further information is available from R. J. Bessey, Associate Director, Institute of Physics, University of Wyoming, Laramie, Wyoming.